OSCILLATOR-GENERATED PERTURBATIONS IN VISCOUS FLUID FLOW AT SUPERCRITICAL FREQUENCIES

E. V. Bogdanova and O. S. Ryzhov

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1. Following [1, 2], we shall use the theory of free interaction [3-5] to study long wavelength perturbations at the inlet to a flat semiinfinite channel. We shall assume that two harmonic oscillators, placed at opposite walls, are the source of the perturbations. In order to give the characteristic frequency of the waves generated by them, we shall introduce the small parameter $\varepsilon = R^{-1/5}$, where the Reynolds number R is calculated according to the width of the channel b^*, the velocity of the flow U^*_∞ at its outlet and the kinematic viscosity ν^*. According to the basic concepts of the theory of free interaction, the frequency $\omega^* = O(\varepsilon U^*_\infty/b^*)$. As far as the location of the oscillators with length $l^* = O(b^*)$ is concerned, we shall assume that they are located at a distance $L^* = O(\varepsilon^{-3}b^*)$ from the inlet.

We shall divide the velocity field on both sides of the central axis of the vessel, which coincides with the x^* axis of a Cartesian coordinate system x^*, y^*, into three regions. In regions 1 and 2, forming the core of the flow, the perturbations have a potential character. Regions 3 and 4 form the main part of the boundary layers at the channel walls and in these regions, the perturbations remain locally nonviscous, but they do contain vortices. The nature of the perturbed motion in the narrow near-wall layers 5 and 6 depends considerably on the viscosity of the liquid; here we can no longer neglect the tangential stresses.

We shall denote the time by t^*, the components of the velocity vector by u^* and v^*, the density by ρ^*, and the pressure by p^*, in which we separate out a constant part p^*_∞. In what follows, it will be useful to combine regions 1 and 2 into a single region 0, assuming that in this region

$$t^* = \varepsilon^{-1}\frac{b^*}{U^*_\infty}t, \quad x^* = b^*\left(\varepsilon^{-3}x_e + x\right), \quad y^* = b^*y_0, \tag{1.1}$$
$$u^* = U^*_\infty[1 + \varepsilon^2 u_0(t, x, y_0) + \ldots],$$
$$v^* = U^*_\infty[\varepsilon^2 v_0(t, x, y_0) + \ldots],$$
$$p^* = p^*_\infty + \rho^* U^{*2}_\infty[\varepsilon^2 p_0(t, x, y_0) + \ldots].$$

Substituting the equations written above into the Navier–Stokes equations for an incompressible fluid gives

$$\partial p_0/\partial x - \partial v_0/\partial y_0 = 0, \ \partial p_0/\partial y_0 + \partial v_0/\partial x = 0, \tag{1.2}$$
$$u_0 = -p_0,$$

from where we conclude that the functions p_0 and v_0 are harmonically conjugate.

As usual in the theory of free interaction, the asymptotic equations for the increments to the parameters of the fluid in regions 3 and 4 are integrated in explicit form [3-5]. We shall omit the corresponding relations, since in what follows we do not need to know them.

We shall introduce the independent variables and the functions sought in layers 5 and 6 next to the walls as follows:

$$t^* = \varepsilon^{-1}\frac{b^*}{U^*_\infty}t, \quad x^* = b^*\left(\varepsilon^{-3}x_e + x\right), \quad y^* = b^*\left(\mp\frac{1}{2} + \varepsilon^2 y_{5,6}\right), \tag{1.3}$$
$$u^* = U^*_\infty[\varepsilon u_{5,6}(t, x, y_{5,6}) + \ldots], \quad v^* = U^*_\infty[\varepsilon^3 v_{5,6}(t, x, y_{5,6}) + \ldots],$$
$$p^* = p^*_\infty + \rho^* U^{*2}_\infty[\varepsilon^2 p_{5,6}(t, x, y_{5,6}) + \ldots].$$

Here, the dimensionless functions satisfy Prandtl's equations

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$$\partial u_{5,6}/\partial x + \partial v_{5,6}/\partial y_{5,6} = 0,\ \partial p_{5,6}/\partial y_{5,6} = 0, \tag{1.4}$$
$$\partial u_{5,6}/\partial t + u_{5,6}\partial u_{5,6}/\partial x + v_{5,6}\partial u_{5,6}/\partial y_{5,6} = -\partial p_{5,6}/\partial x + \partial^2 u_{5,6}/\partial y_{5,6}^2,$$

in which the gradient $\partial p_{5,6}/\partial x$ of the self-induced pressure is found from the conditions for joining expansions (1.1) and (1.3) through intermediate regions 3 and 4.

Let $\lambda = 0.3321$ be a constant that determines the surface friction in the Blasius solution for the boundary layer on a flat plate [6], while $A_3(t, x)$ and $A_4(t, x)$ are arbitrary functions that are proportional to the deviations of the streamlines from horizontal straight lines in the corresponding regions. The boundary conditions obtained in the joining process at $y_0 = \mp 1/2$ give

$$\partial p_{5,6}/\partial x = \partial v_0/\partial y_0,\ v_0 = -\partial A_{3,4}/\partial x. \tag{1.5}$$

In addition, for $y_{5,6} \to \pm\infty$, we have

$$u_{5,6} \mp (\lambda/\sqrt{x_e})y_{5,6} \to \pm(\lambda/\sqrt{x_e})A_{3,4}(t,\ x). \tag{1.6}$$

In all three regions 0, 5, and 6 examined, the perturbations must damp out at infinity upstream.

It remains to write the condition for sticking of the fluid to the surfaces past which it flows. Denoting the amplitude of the oscillations of the oscillators by $\varepsilon^2 ab^*$, we shall give them in the form

$$y^* \pm \frac{1}{2}b^* = \varepsilon^2 ab^* e^{i\omega^* t^*} h_{5,6}\left(\frac{x^* - \varepsilon^{-3}x_e b^*}{b^*}\right).$$

Since the dimensionless frequency $\omega = \varepsilon^{-1}b^*\omega^*/U_\infty^*$,

$$u_{5,6} = 0,\ v_{5,6} = ia\omega e^{i\omega t}h_{5,6}(x) \text{ with } y_{5,6} = ae^{i\omega t}h_{5,6}(x). \tag{1.7}$$

2. Let the amplitude factor satisfy $a \ll 1$. The displacements in the streamlines and the pressure increment can be assumed to be proportional to it, i.e.,

$$(A_{3,4},\ p_{5,6}) = ae^{i\omega t}[A'_{3,4}(x),\ p'_{5,6}(x)]. \tag{2.1}$$

The vertical component $v_0 = ae^{i\omega t}v'_0(x,\ y_0)$ of the velocity in the central region 0, where in view of (1.2) it satisfies Laplace's equation, has an analogous form. We shall expand the functions denoted by the prime in Fourier integrals with respect to the longitudinal coordinate:

$$[\bar{A}_{3,4}(k),\ \bar{p}_{5,6}(k),\ \bar{v}_0(k,\ y_0)] = \int_{-\infty}^{\infty} e^{-ikx}[A'_{3,4}(x),\ p'_{5,6}(x),\ v'_0(x,\ y_0)]dx. \tag{2.2}$$

Denoting the arbitrary constants by b and d, we find $\bar{v}_0 = be^{ky_0} + de^{-ky_0}$. From the boundary conditions (1.5) at $y_0 = \mp 1/2$, it follows that

$$ik\bar{A}_{3,4} = -(be^{\mp(1/2)k} + de^{\pm(1/2)k}),\ i\bar{p}_{5,6} = be^{\mp(1/2)k} - de^{\pm(1/2)k}. \tag{2.3}$$

We shall represent the velocity fields in the near-wall layers 5 and 6 as

$$(u_{5,6} \mp \mu_e y_{5,6},\ v_{5,6}) = ae^{i\omega t}[u'_{5,6}(x,\ y_{5,6}),\ v'_{5,6}(x,\ y_{5,6})], \tag{2.4}$$

where the constant $\mu_e = \lambda x_e^{-1/2}$. Substituting Eqs. (2.4) into the Prandtl equations permits linearizing the latter with respect to the perturbation amplitude a. We shall expand the functions $u'_{5,6}$ and $v'_{5,6}$ defined by the linear equations in Fourier integrals of the form (2.2) and we shall express their transforms $\bar{u}_{5,6}(k,\ y_{5,6})$ and $\bar{v}_{5,6}(k,\ y_{5,6})$ with the help of the relations

$$\bar{u}_{5,6} = -df_{5,6}/dy_{5,6},\ \bar{v}_{5,6} = ikf_{5,6}(k,\ y_{5,6}).$$

The linearization of equations (1.4), derivation of the ordinary differential equations for the functions f_5 and f_6 and integration of the latter follow the outline in [7]. The sticking boundary conditions

$$f_{5,6} = (\omega/k)\bar{h}_{5,6},\ df_{5,6}/dy_{5,6} = \pm\mu_e\bar{h}_{5,6} \text{ at } y_{5,6} = 0 \tag{2.5}$$

follow from (1.7) and, in addition $\bar{h}_{5,6}(k)$ indicate the Fourier transforms of the oscillators $h_{5,6}(x)$ at the lower and upper walls. Let us introduce the complex variables

$$z_{5,6} = \zeta \pm i^{1/3}(\mu_e k)^{1/3}y_{5,6},\ \zeta = i^{1/3}\omega(\mu_e k)^{-2/3}.$$

As a result, we have the derivatives

$$\frac{df_5}{dz_5} = i^{-1/3}k^{-1/3}\mu_e^{2/3}\bar{h}_5 - \mu_e^{-1}\bar{p}_5\left[\frac{dAi(\zeta)}{dz_5}\right]^{-1}\int\limits_{\zeta}^{z_5} Ai(z)\,dz, \qquad (2.6)$$

$$\frac{df_6}{dz_6} = i^{-1/3}k^{-1/3}\mu_e^{2/3}\bar{h}_6 - \mu_e^{-1}\bar{p}_6\left[\frac{dAi(\zeta)}{dz_6}\right]^{-1}\int\limits_{\zeta}^{z_6} Ai(z)\,dz,$$

satisfying the second conditions in (2.5). Integration of Eqs. (2.6) gives the functions f_5 and f_6, which satisfy the first of the conditions indicated.

It remains to satisfy the requirements (1.6) at the outer edges of the boundary layers. In the new variables,

$$df_{5,6}/dz_{5,6} = -\,i^{1/3}k^{-1/3}\mu_e^{2/3}\bar{A}_{5,6} \quad \text{for} \quad |z_{5,6}|\to\infty,$$

from which we derive the relation

$$\bar{A}_3 = -\,\bar{h}_5 + i^{1/3}k^{1/3}\mu_e^{-5/3}\Phi^{-1}(\zeta)\,\bar{p}_5,$$

$$\bar{A}_4 = -\,\bar{h}_6 - i^{1/3}k^{1/3}\mu_e^{-5/3}\Phi^{-1}(\zeta)\,\bar{p}_6, \quad \Phi(\zeta) = \frac{dAi(\zeta)}{dz}\left[\int\limits_{\zeta}^{\infty} Ai(z)\,dz\right]^{-1}$$

between the quantities $\bar{A}_{3,4}$ and $\bar{p}_{5,6}$. Substituting into these relations (2.3), we obtain inhomogeneous linear equations determining the constants b and d:

$$b = -\frac{1}{4}ik\Phi(\zeta)\frac{\left[\Phi(\zeta)\bar{h}_- + i^{1/3}k^{4/3}\mu_e^{5/3}\bar{h}_+\right]\operatorname{ch}\frac{k}{2} - \left[\Phi(\zeta)\bar{h}_+ + i^{1/3}k^{4/3}\mu_e^{5/3}\bar{h}_-\right]\operatorname{sh}\frac{k}{2}}{\left[\Phi(\zeta)\operatorname{ch}\frac{k}{2} - i^{1/3}k^{4/3}\mu_e^{5/3}\operatorname{sh}\frac{k}{2}\right]\left[\Phi(\zeta)\operatorname{sh}\frac{k}{2} - i^{1/3}k^{4/3}\mu_e^{5/3}\operatorname{ch}\frac{k}{2}\right]}, \qquad (2.7)$$

$$d = \frac{1}{4}ik\Phi(\zeta)\frac{\left[\Phi(\zeta)\bar{h}_- - i^{1/3}k^{4/3}\mu_e^{5/3}\bar{h}_+\right]\operatorname{ch}\frac{k}{2} + \left[\Phi(\zeta)\bar{h}_+ - i^{1/3}k^{4/3}\mu_e^{5/3}\bar{h}_-\right]\operatorname{sh}\frac{k}{2}}{\left[\Phi(\zeta)\operatorname{ch}\frac{k}{2} - i^{1/3}k^{4/3}\mu_e^{5/3}\operatorname{sh}\frac{k}{2}\right]\left[\Phi(\zeta)\operatorname{sh}\frac{k}{2} - i^{1/3}k^{4/3}\mu_e^{5/3}\operatorname{ch}\frac{k}{2}\right]},$$

$$\bar{h}_+ = \bar{h}_5 + \bar{h}_6, \quad \bar{h}_- = \bar{h}_5 - \bar{h}_6.$$

3. If the denominator on the right sides of (2.7) is equated to zero, then we obtain the following two dispersion relations:

$$\mu_e^{5/3}\Phi(\zeta) = k(ik)^{1/3}\operatorname{cth}(k/2), \quad \mu_e^{5/3}\Phi(\zeta) = k(ik)^{1/3}\operatorname{th}(k/2) \qquad (3.1)$$

for the frequencies and wave numbers of the free symmetrical and antisymmetrical waves, respectively. We shall briefly present the properties of the roots of (3.1) in the complex k plane with the positive imaginary semiaxis cut out, assuming that the regular branch of the three-valued function $k^{1/3}$ is given by the conditions $-3\pi/2 \leqslant \arg k \leqslant \pi/2$.

If ω is fixed, then there exist four infinite sequences of characteristic wave numbers both for the symmetrical and for the antisymmetrical waves. The first of them is most easily found from the results in [8] for the boundary layer on an isolated plate first on the plane ζ. We shall let $\zeta = \vartheta'|\zeta|^{3/2} \to 0$ and $|\zeta| \to \infty$, while $\vartheta'|\zeta|^{3/2} \to 0$. For any root with number $j \to \infty$,

$$|\zeta j| = [(3\pi/2)(j + 1/4)]^{2/3}. \qquad (3.2)$$

The symmetrical waves are characterized by the equality

$$\vartheta'_{sj} = (-1)^j\frac{\sqrt{2\pi}}{4}\mu_e^{-5/3}\left[\frac{3\pi}{2}\left(j+\frac{1}{4}\right)\right]^{-7/6}k^{1/3}, \qquad (3.3)$$

while the antisymmetrical waves are characterized by the equation

$$\vartheta'_{aj} = (-1)^{j+1}\frac{\sqrt{2\pi}}{4}\mu_e^{-5/3}\left[\frac{3\pi}{2}\left(j+\frac{1}{4}\right)\right]^{-7/6}k^{7/3}. \qquad (3.4)$$

Thus, each of the dispersion relations (3.1) has an infinite sequence of roots kj in the vicinity of the ray arg k = −5π/4 with an accumulation point at the origin.

The presence of three more infinite sequences of roots in the complex k plane is due to the fact that hyperbolic functions enter on the right side of (3.1). Two of these sequences are situated along the edges of the cut arg k = π/2 and arg k = −3π/2, while one is situated

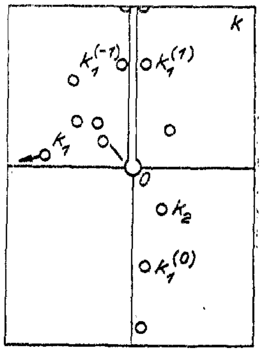

Fig. 1.

in the vicinity of the negative imaginary semiaxis. Let $l = -1, 0, 1$, then the characteristic wave number with number $n \to \infty$ in symmetrical oscillations is determined by the equality

$$k_{sn}^{(l)} = 2(n+1)\pi e^{i\pi(l-1/2)} + \Delta k_{sn}^{(l)},$$
$$\Delta k_{sn}^{(l)} = -2\cdot 3^{2/3}\pi^{-4/3}\Gamma^{-1}(1/3)\,\mu_e^{5/3}(2n+1)^{-4/3}e^{i\pi(1/2-(4/3)l)}, \tag{3.5}$$

while in antisymmetric perturbations, it is given by the equation

$$k_{an}^{(l)} = 2n\pi e^{i\pi(l-1/2)} + \Delta k_{sn}^{(l)},$$
$$\Delta k_{an}^{(l)} = -2\cdot 3^{2/3}\pi^{-4/3}\Gamma^{-1}(1/3)\,\mu_e^{5/3}(2n)^{-4/3}e^{i\pi(1/2-(4/3)l)}. \tag{3.6}$$

Let us now turn to the basic ideas of the linear theory of stability, whose results for long wavelength perturbations with $R \to \infty$ coincide with the results from the theory of free interactions [9, 10]. For this purpose, we shall assume k is a real negative quantity and we shall examine the roots of the dispersion relations (3.1) in the complex plane. The calculations show that for any μ_e, the imaginary part of the first root ω_1 from sequences whose asymptotic form is described by Eqs. (3.2), (3.3), and (3.2), (3.4) changes sign when k passes through some critical value k_*. As mentioned in [2], the critical values themselves $\omega_* < 0$ and $k_* < 0$ can be found by simply rescaling according to the available data for an incompressible boundary layer on an isolated plate. Thus, for symmetrical characteristic waves, $\omega_{s*} = 0.5736$, $k_{s*} = -0.1248$, while for antisymmetrical waves, $\omega_{a*} = 2.9270$, $k_{a*} = -1.4382$ with $\mu_e = 1$. It is significant that $\omega_{s*} < \omega_{a*}$ for any $\mu_e = O(1)$. From the intersections of the curves $\omega_1(k)$ with the abscissa axis it follows that the amplitude of the first mode of both the symmetrical and antisymmetrical waves can become degenerate with time and increase exponentially. The remaining modes turn out to be stable. As far as the roots from the sequences with the asymptotic expressions (3.5) and (3.6) are concerned, they do not have analogs in the ω plane.

Let us examine the inverse Fourier transformations

$$[A'_{3,4}(x),\ p'_{5,6}(x)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ikx}[\bar{A}_{3,4}(k),\ \bar{p}_{5,6}(k)]\,dk \tag{3.7}$$

in the complex k plane with the positive imaginary semiaxis cut out. Equations (2.1) together with (2.3) and (2.7) complete the construction of the streamlines and the pressure increment including their variation with time.

The roots of each of the dispersion relations (3.1) for $\mu_e = O(1)$ are shown schematically in Fig. 1. Varying ω causes a qualitative change in the pattern shown only in one respect: the root $k_1(\omega)$ is displaced from one half-plane into the other. The direction of this displacement with increasing ω follows the arrow. Evidently, Re $k_1 = k_*$, Im $k_1 = 0$ at $\omega = \omega_*$. The root $k_2(\omega)$ is always located in the lower half-plane and, in addition, Re $k_1 = 0$ at $\omega = 0$.

4. Since $\omega_{s*} < \omega_{a*}$, we shall first assume that the frequency of the oscillators $\omega < \omega_{s*}$. The field of the perturbations in the region $x < 0$ extending upstream from the sources is most easily obtained by using the closed contour which includes the arc of a semicircle in the lower half-plane k with a radius extending to infinity. We shall denote the integrand in (3.7) by

$$\Psi(k) = (1/2\pi)e^{ikx}[\bar{A}_{3,4}(k),\ \bar{p}_{5,6}(k)]. \tag{4.1}$$

Based on Jordan's lemma and Cauchy's theorem on residues, we have

$$\int_{-\infty}^{\infty} \Psi(k)\,dk = -2\pi i\,[\mathrm{res}\,\Psi(k_{s2}) + \sum_{n=1}^{\infty} \mathrm{res}\,\Psi\left(k_{sn}^{(0)}\right) + \mathrm{res}\,\Psi(k_{a2}) + \sum_{n=1}^{\infty} \mathrm{res}\,\Psi\left(k_{an}^{(0)}\right). \tag{4.2}$$

The result obtained has a graphic interpretation: perturbations propagate upstream from the oscillators in the form of symmetrical and antisymmetrical Tollmin–Schlichting waves, arising from the roots k_{s2} and $k_{\alpha 2}$ from sequences (3.2), (3.3) and (3.2), (3.4), and two infinite trains of such waves, which are related to roots belonging to sequences (3.5) and (3.6).

In order to study the perturbations in the region $x > 0$, moving downstream beyond the sound sources, we shall use a closed contour, which includes the arc of a semicircle in the upper half-plane k with a radius extending to infinity. We shall denote the edges of the cut along the positive imaginary semiaxis by $\arg k = \pi/2$ and $\arg k = -3\pi/2$ by L_1 and L_2, respectively. As a result

$$\int_{-\infty}^{\infty} \Psi(k)\,dk = -I_1 + I_2 + 2\pi i\left[\sum_{j=1}^{\infty}{}' \mathrm{res}\,\Psi(k_{sj}) \right. \tag{4.3}$$

$$\left. + \sum_{n=1}^{\infty} \mathrm{res}\,\Psi\left(k_{sn}^{(-1)}\right) + \sum_{n=1}^{\infty} \mathrm{res}\,\Psi\left(k_{sn}^{(1)}\right) + \sum_{j=1}^{\infty}{}' \mathrm{res}\,\Psi(k_{aj}) + \sum_{n=1}^{\infty} \mathrm{res}\,\Psi\left(k_{an}^{(-1)}\right) + \sum_{n=1}^{\infty} \mathrm{res}\,\Psi\left(k_{an}^{(1)}\right)\right],$$

$$I_1 = \int_{L_1} \Psi(k)\,dk, \quad I_2 = \int_{L_2} \Psi(k)\,dk,$$

where the prime indicates that the index $j = 2$ is omitted in the summation.

Thus, the nature of the perturbations moving downstream is twofold. The part arising from the residues consists of six infinite sequences of symmetrical and antisymmetrical Tollmin–Schlichting waves, whose parameters are determined by the characteristic functions of the problem of free oscillations. The wave number spectrum of each of the sequences is discrete. The other part of the perturbations, on the contrary, with parameters given by the integrals I_1 and I_2 along the edges of the cut, is characterized by a continuous spectrum.

For $\omega \to \omega_{s*}$, the damping decrement in $\exp(ik_{s1}x)$, entering in res $\Psi(k_{s1})$, becomes arbitrarily small. As a result, the amplitude of the perturbations decreases extremely slowly downstream from the oscillators. When $\omega = \omega_{s1}$, the root $k_{s2} = k_{s*}$ coincides with a point on the negative abscissa axis. In this case, the formal use of the integral transformations leads to the result that it is necessary to subtract πi res $\Psi(k_{s*})$ from the right side of (4.3) and to carry over this quantity with opposite sign to the right side of (4.2). The amplitude of the oscillations is constant along the entire length of the channel with the exception of a small region near the sound sources. If $\omega_{\alpha*} > \omega > \omega_{s*}$, then the root k_{s1} moves into the lower half-plane, as a result of which the term $2\pi i$ res $\Psi(k_{s1})$, which must be subtracted from expression (4.2), disappears entirely from expression (4.3). As long as ω is not too much greater than ω_{s*}, the amplitude of the waves generated increases very moderately upstream from the oscillators.

A similar pattern occurs with further increase in frequency. Indeed, for $\omega \to \omega_{\alpha*}$, the damping decrement in $\exp(ik_{\alpha 1}x)$ from res $\Psi(k_{\alpha 1})$ decreases indefinitely. When $\omega = \omega_{\alpha*}$, the root $k_{\alpha 1} = k_{\alpha*}$ falls on the negative abscissa axis. In addition to the term $2\pi i$ res $\Psi(k_{s1})$, πi res $\Psi(k_{\alpha 1})$ must be subtracted from the right side of (4.3) and both of these quantities should be carried over to the right side of (4.2) with opposite sign. The amplitude of the oscillations remains constant along the vessel upstream from the oscillators and increases exponentially downstream from them. If $\omega > \omega_{\alpha*}$, then the root $k_{\alpha 1}$ also moves into the lower half-plane. In this case, together with $2\pi i$ res $\Psi(k_{s1})$, the term $2\pi i$ res $\Psi(k_{\alpha 1})$ also entirely drops out of expression (4.3), and both terms must be subtracted from expression (4.2). Thus, for $\omega > \omega_{\alpha*}$, both exponentials $\exp(ik_{s1}x)$ and $\exp(ik_{\alpha 1}x)$ already control the increase in the amplitude of the emitted waves.

We must make two remarks concerning the picture of the perturbations given by the integral transformations. First, none of the experiments performed up to now have revealed the sharp increase in the intensity of the signals transmitted upstream, when the frequency of the oscillations passes through one of the critical values ω_{s*} or $\omega_{\alpha*}$. Such data are also

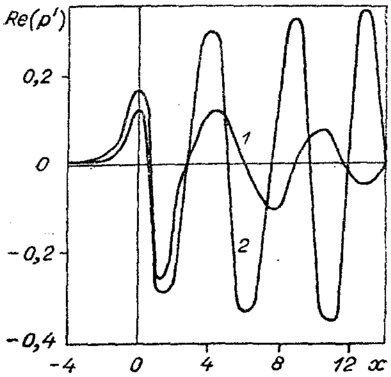

Fig. 2.

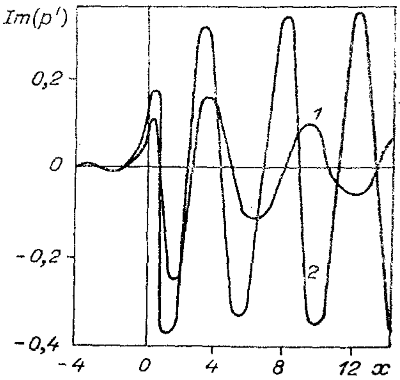

Fig. 3.

absent for the degenerate oscillations in the boundary layer and developed viscous flow in a channel or pipe. Second, for $\omega > \omega_{s*}$ and especially $\omega > \omega_{a*}$ the solution of the linear problem being studied must be sought among a class of functions increasing exponentially along the longitudinal coordinate x in order to obtain the observed increase in the amplitude of the perturbations downstream along the flow, which gives rise to turbulent pulsations. Naturally, for the class indicated, there are no theorems guaranteeing the uniqueness of the solution. However, as pointed out in [11], the classical solution with the field of perturbations becoming degenerate with $x \to \pm\infty$ is unacceptable from a physical point of view.

In order to ensure a unique choice of solutions at supercritical oscillator frequencies, an additional postulate is necessary. As experiments show, no sudden changes (at least at moderate distances from the sources) in the fields of the perturbations occur when ω reaches one of the critical values ω_{s*} and ω_{a*}. For this reason, we shall require that the solution of the linear problem given by Eqs. (4.2)-(4.3) be continuous with respect to ω for any finite x. This requirement indicates that relations (4.2) and (4.3) remain in force for any $\omega_{a*} \geqslant \omega \geqslant \omega_{s*}$ and $\omega > \omega_{a*}$. All boundary conditions of the problem will be satisfied, since the use of Eqs. (4.2) and (4.3) at supercritical frequencies is allowed by the possibility of adding to the solution constructed with the help of integral transformations characteristic functions increasing exponentially with $x \to \infty$. The residues res $\Psi(k_{s1})$ and res $\Psi(k_{a1})$ give the characteristic functions sought, which represent the first mode of the symmetrical and antisymmetrical Tollmin–Schlikhting waves.

The rule presented above gives a continuous evolution of linear perturbations with respect to ω in a region where there are no turbulent pulsations developing at some distance from the sources. Of course, it has a universal character and is applicable equally to the boundary layer on a plate and developed viscous flow in a channel or pipe.

Calculations of the real and imaginary parts of the excess pressure for identical oscillators oscillating in phase are presented in Figs. 2 and 3, respectively. It was assumed that the parameter $\mu_e = 0.97$, while $h_s(x) = h_a(x) = h(x)$ and, in addition,

$$h(x) = \begin{cases} 0, & x \leqslant 0, \\ x, & 0 \leqslant x \leqslant 1, \\ 2 - x, & 1 \leqslant x \leqslant 2, \\ 0, & 2 \leqslant x, \end{cases}$$

from where we obtain the Fourier transform $h(k) = -(1 - e^{ik})^2/k^2$. In this case, only antisymmetrical perturbations are excited, for which Eqs. (2.7) reduce to

$$b = d = \frac{1}{2} ik\Phi(\zeta) \frac{h(k)}{\Phi(\zeta)\,\mathrm{ch}\,\frac{k}{2} - i^{1/3}k^{4/3}\mu_e^{5/3}\,\mathrm{sh}\,\frac{k}{2}}.$$

The number 1 indicates waves with subcritical frequency $\omega = 2 < \omega_{a*}$ and the number 2 indicates waves with supercritical frequency $\omega = 2.92 > \omega_{a*}$. We recall that for $\mu_e = 0.97$, the critical frequency of free antisymmetrical perturbations $\omega_{a*} = 2.81$. Equations (4.1)-(4.3) were used in the calculations both for subcritical and supercritical regimes. Upstream from the oscillators, the signals emitted damp out extremely rapidly and a small intensification of the wave process occurs downstream from the sources at frequency $\omega = 2.92$.

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STATISTICAL CHARACTERISTICS OF A PASSIVE ADMIXTURE IN A HOMOGENEOUS ISOTROPIC TURBULENCE FIELD

I. V. Nikitina and A. G. Sazontov

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1. It is well known that in studying different physical phenomena, in particular in order to understand mixing processes, it is necessary to know the spectral characteristics of the passive admixture located in a developed turbulence field [1]. Information on the statistical properties of the corresponding scalar fields (concentration, temperature, moisture content and so on) is important in analyzing the propagation and scattering of acoustical, optical, and radio waves in a turbulent medium [2].

In this paper, we study the spectral structure of a passive admixture with the help of a regular procedure, based on Wyld's diagrammatic technique [3]. Using improved approximations of direct interactions, we find the spectrum of the passive impurity in the inertial-convective interval, obtained previously from dimensional considerations [4, 5] and semiempirical theories, which are reviewed in [6, 7]. The flow direction of the passive admixture is determined from the scale spectrum. The asymptotic behavior of the spectrum is studied in the viscodiffusion interval of wave numbers. For generality of the presentation, the spectral characteristics are analyzed in a space with arbitrary dimensionality d.

2. In order to describe the passive admixture in the homogeneous isotropic turbulence field, we shall examine the Navier–Stokes equations, the equation of continuity, and the diffusion equation, which in the **k** representation have the form

$$\left(\frac{\partial}{\partial t}+\nu k^2\right)v_{\mathbf{k}}^{\alpha}=-\frac{i}{2}P_{\mathbf{k}}^{\alpha\beta\gamma}\int v_{\mathbf{k}_1}^{*\beta}v_{\mathbf{k}_2}^{*\gamma}\delta^{(d)}(\mathbf{k}+\mathbf{k}_1+\mathbf{k}_2)\,d^{(d)}\mathbf{k}_1d^{(d)}\mathbf{k}_2,$$

$$k_\alpha v_{\mathbf{k}}^{\alpha}=0,$$

$$\left(\frac{\partial}{\partial t}+\chi k^2\right)\vartheta_{\mathbf{k}}=-ik_\alpha\int v_{\mathbf{k}_1}^{*\alpha}\vartheta_{\mathbf{k}_2}^{*}\delta^{(d)}(\mathbf{k}+\mathbf{k}_1+\mathbf{k}_2)\,d^{(d)}\mathbf{k}_1d^{(d)}\mathbf{k}_2,$$

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